

AP Calculus BC

Integration by Parts - #1

1) $\int x \sin(6x) dx$

$u = x \quad v = -\frac{1}{6} \cos(6x)$
 $du = dx \quad dv = \sin(6x) dx$

$-\frac{1}{6} x \cos(6x) + \frac{1}{6} \int \cos(6x) dx$
 $-\frac{1}{6} x \cos(6x) + \frac{1}{36} \sin(6x) + C$

2) $\int 9t e^{2t}$

D	I
$9t$	e^{2t}
9	$\frac{1}{2} e^{2t}$
0	$\frac{1}{4} e^{2t}$

$\frac{9}{2} t e^{2t} - \frac{9}{4} e^{2t} + C$

3) $\int x^2 e^{10x} dx$

D	I
$+ x^2$	e^{10x}
$- 2x$	$\frac{1}{10} e^{10x}$
$+ 2$	$\frac{1}{100} e^{10x}$
0	$\frac{1}{1000} e^{10x}$

$\frac{1}{10} x^2 e^{10x} - \frac{1}{50} x e^{10x} + \frac{1}{500} e^{10x} + C$

4) $\int x^2 \cos(4x) dx$

D	I
$+ x^2$	$\cos 4x$
$- 2x$	$\frac{1}{4} \sin 4x$
$+ 2$	$-\frac{1}{16} \cos 4x$
0	$-\frac{1}{64} \sin 4x$

$\frac{1}{4} x^2 \sin 4x + \frac{1}{8} x \cos 4x - \frac{1}{32} \sin 4x + C$

5) $\int 20x^2 \ln x dx$

$u = \ln x \quad v = \frac{20}{3} x^3$
 $du = \frac{1}{x} dx \quad dv = 20x^2 dx$

$\frac{20}{3} x^3 \ln|x| - \frac{20}{3} \int x^2 dx$

$\frac{20}{3} x^3 \ln|x| - \frac{20}{9} x^3 + C$

6) $\frac{dy}{dx} = 9x \csc^2(3x) \quad y=1 \quad x = \frac{\pi}{2}$

D	I
$9x$	$\csc^2(3x)$
9	$-\frac{1}{3} \cot(3x)$
0	?

$-3x \cot(3x) + \int 3 \cot(3x)$
 $y = -3x \cot(3x) + \ln|\sin(3x)| + C$
 $1 = -3\left(\frac{\pi}{2}\right) \cot\left(\frac{3\pi}{2}\right) + \ln|\sin(3\pi/2)| + C$
 $1 = C$
 $y = -3x \cot(3x) + \ln|\sin(3x)| + 1$

$$7) \int 6x^3 e^{-5x} dx$$

$\frac{D}{+6x^3}$	$\frac{I}{e^{-5x}}$	
$-18x^2$	$\rightarrow -\frac{1}{5}e^{-5x}$	
$+36x$	$\rightarrow \frac{1}{25}e^{-5x}$	
-36	$\rightarrow -\frac{1}{125}e^{-5x}$	
0	$\rightarrow \frac{1}{625}e^{-5x}$	

$$-\frac{6x^3}{5}e^{-5x} - \frac{18x^2}{25}e^{-5x} - \frac{36x}{125}e^{-5x} - \frac{36}{625}e^{-5x} + C$$

8)

$$\int_0^{\pi/2} (x-2) \sin x dx$$

$\frac{D}{+x-2}$	$\frac{I}{\sin x}$
-1	$-\cos x$
0	$-\sin x$

$$-(x-2)\cos x + \sin x \Big|_0^{\pi/2}$$

$$\left[-\left(\frac{\pi}{2}-2\right) \cdot 0 + 1 \right] - [2] = -1$$

9)

$$\int_1^{e^2} x^5 \ln(x) dx$$

$$u = \ln x \quad v = \frac{1}{6}x^6$$

$$du = \frac{1}{x} dx \quad dv = x^5 dx$$

$$\frac{1}{6}x^6 \ln x - \frac{1}{6} \int x^5 dx$$

$$\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 \Big|_1^{e^2}$$

$$\left(\frac{1}{3}e^{12} - \frac{1}{36}e^{12} \right) + \frac{1}{36}$$